PB-228 058

DELAWARE ESTUARY COMPREHENSIVE STUDY. TECHNICAL REPORT NUMBER 2. ESTUARINE WATER QUALITY MANAGEMENT AND FORECASTING

Public Health Service
New York, New York

1962

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

PB 228 058

Delaware Estuary Comprehensive Study

Report Number 2

[2] Division of Water Supply and Pollution Control
Region II , New York, New York

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
U S Department of Commerce
Springfield VA 22151

[1] U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Public Health Service

DELAWARE ESTUARY COMPREHENSIVE STUDY

TECHNICAL REPORT NO. 2

Estuarine Water Quality Management and Forecasting

This Technical Report, one of a series prepared by the staff of the Delaware Estuary Comprehensive Study is for the information of interested agencies and individuals. Reports will be added to the series as developments from the various phases of the Study provide additional information that will be of value to others concerned with water pollution control.

Technical Report

No. 1 Report on Analysis of the Dye Diffusion Data in the Delaware River Estuary - Evaluation of Diffusion Coefficients

DELAWARE ESTUARY COMPREHENSIVE STUDY

TECHNICAL REPORT NO. 2

Estuarine Water Quality Management and Forecasting[1]

SYNOPSIS

Techniques are described for the forecasting and optimum management of water quality in the estuarine environment. The understanding of water quality variations can be improved through the use of a systems analysis approach. The results of this analysis can then be used in programming problems to determine the optimum allocation of funds for several significant water quality control problems. Examples are presented of the nature of estuarine quality changes and the results of a systems analysis applied to the Delaware estuary are given.

Introduction

This paper considers the problem of estuarine water quality management and forecasting as one of making rational decisions that will affect water quality in the light of the understanding of the physical, social and economic environment. While the understanding may be less than complete, this does not preclude a management scheme which is in some sense commensurate with a given level of knowledge.

---

[1]Presented at ASCE Water Resources Conference, Milwaukee, Wisconsin, May 15, 1963.

As will be seen, the analysis as described in this paper presupposes the existence of continuous or quasi-continuous data on water quality and the inputs that affect quality as well as reliable and accurate data on the costs of undertaking certain control steps. With this data available, some novel methods will be discussed which lead to a better understanding of the estuarine environment and its effects on quality. This phase is made somewhat more complex by the hydrodynamic, chemical and biological characteristics of estuaries. There are many difficulties associated with oscillatory motions, gradients of salinity and marked changes in biological species. Nevertheless, a level of insight can be obtained which will afford water quality administrators and managers with techniques for computing future quality conditions. This extension of knowledge of the environment can then be applied in such a way that decisions can be made with more confidence than was previously possible.

Causal Analysis in the Estuarine Environment

Deterministic responses

The time and space variations of water quality in an estuarine system may be considered as the general problem of quality forecasting. Marine waters are subjected to several additional forces such as tidal and pressure gradient forces that are not usually found in fresh water environments. If an approach is presented for the analysis and forecasting of water quality in the estuarine environment, then the problem of describing quality in rivers or lakes assumes the role of a special case. For example, an analysis of a general problem which includes tide induced turbulence as a diffusion mechanism can be readily particularized to a non-tidal river where this diffusive phenomenon is simply assumed to be negligible. Furthermore, analyses for three dimensional situations are of course easily specialized to the case of one-dimensional non-tidal flow in rivers. The estuarine environment although complex embodies the general water quality problem and once questions are posed validly in the marine system, they can be subsequently particularized to other situations with a minimum of difficulty. The following discussion then presents some concepts for analyzing and forecasting estuarine water quality. Nevertheless, the analyses can be applied to other bodies of water such as lakes and non-tidal rivers.

A fundamental concept involved in an analysis and forecasting situation is the presence of a series of successive cause and effect relationships which require that a series of input-output determinations be made. This presupposes a knowledge of how the environment acts on an input to produce an output. An insight of this action is gained by using significant periodicities in a time record of a variable at a particular point in an estuary. Figure 1 presents

the average daily variation of dissolved oxygen in the Delaware estuary at Philadelphia. This record obtained by monitoring stations operated jointly by the City of Philadelphia and the U.S.G.S. was constructed by digitizing a continuous trace at hourly intervals and subsequently computing 24 hour means. The presence of a strong annual period is obvious and indicates some interrelationship between a causative variable with an annual period and the DO. A second, more striking, example is given in Figure 2 which indicates the variation of average daily water temperature for the same time period and location. The annual period accounts for the largest portion of the variation and acts as one of the predominant causes in the annual variation of DO.

For each water quality variable, an examination of direct inputs affecting the variable is important. Furthermore, a distinction between direct inputs and those factors affecting the physical distribution of a quality variable throughout the estuary is desirable. For example, a listing of those factors which affect DO through chemical, biochemical, or biological mechanisms would include:

a) Water temperature, which acts to determine the rate of oxygen utilization, the level of saturation of oxygen and the photosynthetic production of oxygen;

b) Amount of organic matter present which would utilize oxygen; and

c) The amount of solar radiation and nutrients which results in photosynthesis.

These inputs or causative functions if acting in a quiescent body of water would result in some specific variation of DO. The DO time variation would depend solely on the time variation of the inputs and any spacial variations which would be present would exist only because temperature may be higher

in one section than in another. However, in a moving turbulent estuary, the mechanism of diffusion and advection (or transport) of DO would also act on the inputs, distribute them throughout the system and result in additional time and space variations. The distinction may then be made between direct "forcing functions" which "force" the water quality variable to respond and hydrodynamic functions which determine to what extent the quality at one point will depend on forcing functions at another point.

State vectors

Once the preceding distinction is made, the problem of describing and ultimately forecasting water quality can be viewed in the light of a systems analysis approach. The basic concept of a water quality system is embodied in the notion of the "state" of the environment at any time (Bellman[2]). This may be written in terms of a state vector,

$$\begin{pmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \\ \cdot \\ \cdot \\ \cdot \\ c_n(t) \end{pmatrix}$$

where $c_i(t)$ is the water quality at time t for section i of the estuary. A description of the spacial variation of quality in an estuary is then given by a knowledge of the state vector. From a control or management engineering viewpoint, it is not necessary to describe the process for a continuous time stream nor is it necessary to describe the process at all

---

[2] Adaptive Control Processes: A Guided Tour by R. Bellman, Princeton University Press, 1961

points in space. There are limitations on the extent of the quality description or forecast that can be made in either the space or time domain. These limitations depend primarily on the limits of control of the inputs. This will be discussed more fully below.

With a water quality state vector defined, it is natural to continue and introduce the state vector for the forcing functions of the system, i. e.

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \\ \vdots \\ f_n(t) \end{pmatrix}$$

where $f_i(t)$ represents those inputs that force a particular water quality variable to respond. For the case of dissolved oxygen, the $f_i(t)$ may take the form $f_i(t) = r\, c_{si} - d_i\, L_i + P_i$

where r_i = reaeration coefficient

c_{si} = saturation value of DO

d_i = decay coefficient

L_i = BOD

and P_i = all other sources and sinks of oxygen including photosynthesis, chemical oxygen demand, etc.

It is seen that for this particular case, each $f_i(t)$ is composed of several sub-forcing functions, the algebraic sum of which enters into the environmental system. As a second example, the forcing function vector may be applied to the description of longitudinal salinity distribution in an estuary. If $i = 1$ is the head of the estuary and $i = n$ is the ocean, then under the assumption of a completely fresh water inflow and no differences in evaporation and precipitation as one proceeds along the estuary, the only source of salt would of course be from the ocean. The vector then may be written as

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ f_n(t) \end{pmatrix}$$

The relationship between the quality state vector and the forcing function vector is contained in the definition of a "system" namely, the transformation of the forcing function vector into the quality state vector through the mechanisms of the immediate and surrounding environment. This requires the introduction of a transfer matrix and the resulting transformation may be written in the following form

(1) $$(T)\,(c) = (f)$$

where (c) = the state vector

(f) = the forcing function vector

and (T) = a transfer matrix

It is interesting to note that equation (1) is sufficiently general at this point to incorporate a larger area of interest. The state vector, for example, may represent the quality (or quantity) of water at different locations $i = 1, 2, \ldots n$ spaced throughout an entire river basin. The forcing function vector may embody the industrial and municipal character of the areas of the basin and hence the potential for waste discharges (or water uses). The transfer matrix in this context then would represent the effects of a changing forcing function vector on the state vector through either obvious physical interrelationships, e. g. the hydrodynamic and atmospheric environments or through socio-economic relationships namely, work force and tourist migrations between areas of the region. The areas might represent sub-drainage basins of several hundred square miles of the larger basin and the time intervals of concern be on the order of years. Under time and space scales of this size, detailed descriptions of day to day or mile to mile phenomena are not useful since attempts at description are usually made for purposes of overallmanagement or surveillance. Large area and long time descriptions imply management measures of the same scale. The nature of the forcing functions as indicated above is such as to preclude any effective use of "short" term (i.e. day to day or week to week) management procedures. As the time and space scales under consideration become more and more compressed (or expanded) the set of possible control procedures changes from broad-scale guidance of development to smaller scale controls on existing facilities.

Therefore, although a system can be defined in general mathematical and linguistic terms, it is tacitly understood that the user of the definition has in mind some particular scale of description and associated management

procedures. It is with this scale that one proceeds then to make more definitive the expression given by equation (1). In this paper, equation (1) will be particularized to a description of the water quality in a one-dimensional estuary and subsequent use of the description for forecasting purposes. Only the general notions will be expressed. A more complete mathematical treatment is given by Thomann[3].

The one-dimensional estuary

If a one-dimensional estuary is segmented into n segments, the rate of change of a quality variable, $c_k(t)$ in segment k will depend on

a) the net volume transport of the variable from k-1 into k and from k into k + 1

b) an exchange mechanism due to the tidally induced turbulence from both k-1 and k+1 into k

c) sources and sinks of the variable $c_k(t)$ acting in segment k.

The differential equation which combines these phenomena is given by

$$V_k \frac{dc_k}{dt} = Q_{k-1,k} \left(\frac{c_{k-1}+c_k}{2}\right) - Q_{k,k+1} \left(\frac{c_k + c_{k+1}}{2}\right) + E_{k-1,k} (c_{k-1} - c_k) + E_{k,k+1} (c_{k+1} - c_k) + V_k f_k (t) \quad (2)$$

where V_k = volume of segment k

$Q_{k-1,k}$ = net flow from segment k-1 to k

$E_{k-1,k}$ = an eddy exchange coefficient from k-1 to k

f_k = all forcing functions acting in segment k which cause either an increase or decrease in $\frac{dc_k}{dt}$

---

[3] A Mathematical Model for Dissolved Oxygen, by Robert V. Thomann, Submitted for publication, Proceedings Journal of Sanitary Engineering, ASCE

The assumptions involved in writing equation (2) are

a) the rate of change of c_k is affected by the average concentration between k and adjacent segments both upstream and downstream

b) the eddy exchange phenomenon proceeds directly proportional to the difference of the concentrations

c) Q and E are constants

If now $Q_{k-1,k} = Q_{k,k+1} = Q$ (no incremental inflow in section k)

and $E_{k-1,k} = E_{k,k+1} = E$, a collection of terms in equation (2)

gives $(-Q-E)c_{k-1} + (V_k \frac{d}{dt} + 2E)c_k + (Q-E)c_{k+1} = V_k f_k(t)$

or

(3) $$a_{k-1,k}\, c_{k-1} + a_{kk}\, c_k + a_{k+1,k}\, c_{k+1} = f_k(t)$$

where $a_{k-1,k} = -Q - E$

$a_{kk} = V_k \frac{d}{dt} + 2E$

$a_{k+1,k} = Q - E$

and the volume V_k has been absorbed into $f_k(t)$

Special conditions are needed for $k = 1$, and $k = n$, the end points of the estuary. For these segments, it may be assumed, for example, that eddy diffusion does not continue past the head of tide and that at the ocean, there is an infinite reservoir of the variable $c(t)$.

For each of the n section, an equation similar to the differential equation (3) can be written. The final result then is the set of equations given by

$$
\begin{aligned}
& a_{11}c_1 + a_{12}c_2 + 0 + \ldots + 0 = f_1(t) \\
& a_{21}c_1 + a_{22}c_2 + a_{23}c_3 + 0 + \ldots + 0 = f_2(t) \\
(4)\quad & 0 + a_{32}c_2 + a_{33}c_3 + a_{34}c_4 + 0 + \ldots + 0 = f_3(t) \\
& \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad \vdots \\
& 0 + \qquad\qquad . \qquad\qquad + a_{nn}c_n = f_n(t)
\end{aligned}
$$

Equation (4) can also be written in matrix notation.

$$
(5)\quad \begin{pmatrix} a_{11} & a_{12} & 0 & \cdots & 0 \\ a_{21} & a_{22} & a_{23} & 0 & \cdots \\ 0 & a_{32} & a_{33} & a_{34} & \cdots \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & & a_{nn} & \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_n \end{pmatrix}
$$

or $$(A)(c) = (f)$$

The similarity between equations (1) and (5) is now evident. Hence, the transfer matrix for the one dimensional estuary model incorporates the major hydrodynamic features of the estuary via the elements a_{ij}. As shown in equation (3), these elements include the net flow between sections, the volume of the section and the eddy exchange mechanism. The solution for the state vector is given by

$$(6)\qquad (c) = (A)^{-1}(f) = (\emptyset)(f)$$

where $(\emptyset) = (A)^{-1}$ transforms (f) into (c).

Equation (6) indicates that the state of quality in the estuary at any time is given in terms of the interrelationships between the forcing function vector and the hydrodynamic regime. More specifically, for a given set of forcing functions the transfer matrix indicates the effect of the estuary proper in diffusing and transporting the forcing function into different sections of the estuary resulting finally in a state of quality.

The quality in any section j due to a forcing function in section i is

then given by

$$c_j(t) = \phi_{ij} f_i(t) \tag{7}$$

where ϕ_{ij} is a transfer function.

The total response in j due to all functions is given by a linear superposition

$$c_j(t) = \sum_i \phi_{ij} f_i(t) \tag{8}$$

If in equation (7) the forcing function is given by a periodic relationship, i. e.

$$f_i(t) = f_{oi} \cos(\omega t)$$

where f_{oi} = amplitude of f_i

$\omega = \frac{2\pi}{T}$ = angular frequency

T = period of forcing function

then as indicated by Thomann[3], the response can be computed from

$$c_j(t) = \sum_i A_{ij}(\omega) f_{oi} \cos(\omega t + \theta_{ij}(\omega)) \tag{9}$$

where $A_{ij}(\omega)$ = an amplitude attenuation factor

$\theta_{ij}(\omega)$ = a phase shift

Both A_{ij} and θ_{ij} can be determined from the transfer matrix. Equation (9) indicates that a knowledge of the hydrodynamic characteristics of the body of water leads to a knowledge of how the estuary will attenuate the magnitude and shift the phase of an input forcing function. It is noted that A and θ depend on frequency but are independent of the magnitude of the input forcing function. It should also be noted that the input function is not restricted to periodic phenomena. The analysis as outlined above for an harmonic input can be generalized to

include almost-periodic, transient or random inputs. This has particular value since several of the more important inputs are of a short term transient nature. In all cases, the water quality response to different time varying inputs can be obtained if the transfer functions are known.

The procedure outlined above was used to compute transfer functions for a portion of the Delaware estuary. In this case, it was desired to verify the observed DO time series given in Figure 1 at Pier #11 North in Philadelphia, Pa. A nine-section stretch of the estuary was used with the desired response placed in section #7. This allowed for two sections downstream and six sections upstream. Hence, in order to compute the response in #7 (the location of the observed series) nine transfer functions were needed. Each function, $\emptyset_{i7}$, represents the effect of forcing functions in each section i, (i = 1, 2 ... 9) on the response in section #7. The sections were not chosen in a symmetrical fashion around the response section since it was anticipated that forcing functions downstream from the section under consideration would have considerably less effect than forcing functions in upstream sections.

In order to determine the change in the transfer function with changes in the hydraulic regime, three different levels of fresh water flow were used as well as two different levels of eddy diffusion. Since the general equations were applied to a description of the dissolved oxygen variations, an additional parameter, namely the reaeration coefficient enters the transfer function. This results from the inclusion of the DO saturation forcing function which includes the product of the reaeration coefficient and the dissolved oxygen.

The eddy exchange coefficient is comparable to the classical diffusion coefficient and may be computed by equating the steady state salt distribution in the finite case with the distribution in the infinitesmal case. This leads to an expression relating the diffusion and exchange coefficients.

(10) $$E_{i,i+1} = \frac{K_{i,i+1} \; A_{i,i+1}}{0.5 \; (l_i + l_{i+1})}$$

where $K_{i,i+1}$ = diffusion coefficient

$A_{i,i+1}$ = cross sectional area

$l_{i,i+1}$ = length of section

This equation can then be used to estimate E if estimates of K based on the diffusion-advection equation are available.

O'Connor[4] has provided estimates for the longitudinal eddy diffusion coefficients for the Delaware estuary. He obtained values ranging from 2 - 5 mi^2/day for the length of the estuary under consideration. Transfer functions were computed using two values of E corresponding to diffusion coefficients of 2 and 5 mi^2/day.

Table 1 indicates the different sets of parameters used to compute the different transfer functions.

Table 1

System Parameters Used to Compute

Transfer Functions

($r = 0.2\ day^{-1}$)

Set No.	Q (cfs)	K mi^2/day
1	8	2
2	8	4
3	10	2
4	10	4
5	12	4

As noted above, each set results in nine transfer functions for a total of 45 functions.

Figure 3 presents some of the results of the analyses of the transfer functions for set #1. The first subscript indicates the section in which the forcing

---

[4] Oxygen Balance of an Estuary by Donald J. O'Connor, Journal of Sanitary Engineering Div., Proc. ASCE, vol. 86, No. SA3, pp. 641-684, 1960.

function is applied while the second subscript indicates the section in which the DO response is to be computed. The abscissa is in terms of integral values of a basic frequency of 2π n/365. Hence, the value 365 on the abscissa corresponds to a frequency of 2π or a period of one day. It can be noted that A_{77}, i. e. the amplitude response in the same section as that in which the forcing function is applied, decreases with increasing frequency indicating that forcing functions of high frequency (periods of approximately less than a day) will be damped and the response will be close to zero. Furthermore, as forcing functions are applied in sections further upstream, i. e. for $i<7$, A_{i7} decreases over all frequencies and in the limit approaches zero.

The phase shift for responses in the same section approaches 90° asymptotically and for low frequency inputs is close to zero. As is expected, for a fixed frequency the phase shift increases as inputs upstream are considered. In all cases, the phase angle is close to zero for low frequency inputs which indicates that the maximum response will occur at about the same time as the input.

Figure 4 indicates the effect of doubling the diffusion coefficient while the flow and reaeration parameters are held constant. The amplitude attenuation increases with increased mixing since input variability would be lessened due to a higher degree of turbulence. It is interesting to note the change in the shape of the curves at the higher level of diffusion. It is not immediately apparent whether there is any physical cause for the change in the shape. There is no doubt, however, that it is real and not due to computational difficulties.

The effect of different flow levels is shown in Figure 5 and as can be seen, increased attenuation results from increased flow. This is to be expected since higher flows will tend to increase the advection out of the section and will

transmit inputs more rapidly then under lower flow regimes.

The above examples indicate the general shapes of the amplitude attenuation and phase shifts with increasing frequency and with forcing functions in different locations. These transfer functions were computed from the basic differential equations and can be used to operate on a known input to produce an output quality.

In some cases, however, the system may be so complex that the construction of an empirical transfer function may be the most advantageous procedure. Formally, this can be obtained by solving equation (7) for the transfer function, i. e.

$$\phi_{ij} = \frac{c_j(t)}{f_i(t)} \tag{11}$$

It is seen that this requires information on the output and input simultaneously, the quotient of which yields an empirical transfer function. This function may then be used in future similar situations and can be made to operate on the new input. The empirical transfer function is useful especially in describing transient situations where the theoretical mechanisms are not entirely clear. For example, a transient wind field directing a force up an estuary may produce a sharp transient increase in salinity. If data are available on the input wind field and the output salinity variation, the transfer function can then be computed. For future wind fields of a similar type, this empirical function can then be used to forecast the transient salinity pattern.

Forecasting water quality

The above systems analysis approach affords a means for forecasting levels of water quality on both a short term and long term basis. If adequate representations of the transfer functions and the forcing functions are at hand, the ability exists to generate a quality output for any section of the estuary and for different time periods. Further, since all of the input forcing functions

produce independent responses, one can impose inputs in certain sections and note the responses in other sections independently of any other influences. A good forecasting scheme then depends on the extent of knowledge of how the transfer function behaves and on adequate representations of the inputs. As pointed out previously, this latter requirement may be a combination of periodic, transient and random phenomena. The relative weight of these characteristics of the inputs can be determined by means of classical Fourier and random process analyses.

It is obvious that the type of forecast (i. e. the time spacing of the predictions) depends somewhat on the managerial problems to be solved. For example, it would do little good to attempt to develop a model which would provide forecasts of conditions every hour. A sufficient amount of control on the environment could not be exercised in such a short time. The model as outlined above is not capable of describing phenomena on a scale less than a tidal cycle since the advection and eddy exchange terms are considered as net processes over a tidal cycle. Indeed, it is felt that even average daily values of water quality is a fine scale description in view of the considerable ranges in quality which occur from day to day throughout the year, e. g. Figures 1 and 2.

As mentioned previously, the model was used to verify the DO time series shown in Figure 1. Forcing function models were constructed and included the following:

a) effects of temperature on saturation value, decay coefficient and photosynthetic production

b) effects of sunlight intensity in photosynthetic production

c) effects of waste discharges in terms of ultimate BOD

The periodic portions of these inputs were then used in conjunction with the computed transfer functions to generate a computed DO time series. This series was composed of many individual periodic components, the linear superposition of which provided the total periodic DO response. As is evident from inspection of the record, a transient response occurred in late February. In order to incorporate this transient phenomenon into the verification, an analysis was carried out which makes use of a "weighting" function in the time domain analogous to the transfer function in the frequency domain. A description of the computation of this weighting function is given in Thomann[3] and the use of the function in a managerial problem is discussed under a subsequent section of this paper, Problems of Tactics. An analysis of input functions during this transient state indicated that a sharp increase in flow was probably the major factor although it appears that a transient increase in sewage effluent was also significant. A sufficient amount of basic data was not available during this transient stage. However, a rational procedure would indicate that an increase in flow would essentially result in a transient decrease in the BOD sink for the area of the desired verification. An input transient was then constructed on the basis of this reasoning and was fed into the model. The weighting function then operated on the input to produce the transient output, which was then superimposed on the periodic portion of the output to produce the total response shown in Figure 6.

The agreement between these curves is quite good and generally describes the overall features of the DO time variation. The presence of the higher frequency terms in the computed curve is due primarily to non-linear effects in some of the inputs. It is these higher frequency phenomenon (i. e. periods

less than 365 days) which produce the leveling of the DO during the summer months which generally agrees with what was observed. However, there are several discrepancies. During mid-spring the curve began to drop about 10 days too soon and during late summer it began to rise about two months earlier than the observed variation. In order to have the computed curve make the sharp almost right angled turn in mid-Noyember, it would be necessary either to include several higher harmonics of the input or to impose a suitable input transient. One possible explanation for the discrepancy is the lack of inclusion of higher frequency terms in the BOD input. In the verification analysis only the mean and first harmonic of the BOD were included due to the lack of sufficient field data to detail the presence of higher frequency terms. Presumably, better field data from which these terms could be estimated would improve the verification. In an actual forecasting situation, however, the September-October difference would not be allowed to persist. This can be accomplished by a continual feedback of a comparison between forecasted and observed values. Under this condition, the forecast would be continually revised in the light of the new knowledge which was received.

Control of inputs affecting estuarine water quality

In any control engineering system, the final desired accuracy of the forecasts depends to a large extent on the limitations of controlling the inputs. These limitations in turn depend on the magnitude of the variability of the inputs as well as the nature of the control process. In an estuarine system, there are several points where water quality control can be exercised. These can conveniently be grouped into two categories, control of the natural environment and control of the man-made environment. No attempt is made here to determine the engineering or economic feasibility of the controls. They are listed

as possible alternatives and it is not at all clear whether any one should be eliminated on an engineering or economic basis. Controls in the natural environment include

a) fresh water inflow regulation

b) changing channel regimes and scheduling of dredging operations

c) tidal barriers

d) changing water temperature

e) possible changes in degrees of turbulence

f) management of photosynthetic production

Controls in the man-made environment include

a) control of municipal and industrial waste effluents

b) management of water withdrawals

c) in-plant control of quality

As indicated, the extent of control that can be exercised on the above factors depends on the variability and nature of the process. For example, the control of municipal effluent organic load is complicated by two factors, a) the variability of the influent into a sewage treatment plant and b) the complexity of the treatment process which does not allow a precise measure of quality control at each of the treatment units. An example of the variability of a sewage plant effluent is shown in Figure 7 which represents the average daily load discharged into a constant volume section of the Delaware estuary. From a managerial veiwpoint, high frequency almost random inputs of this type implies the necessity for provisions of a high degree of flexibility in the facilities. in order to exercise a higher degree of control.

Predominantly low frequency inputs, e. g. annual and longer changes in effluent load or incoming fresh water flow, implies the need for investments which

will result in the improvement of the annual mean water quality. The optimum allocation of funds for controlling both the long and short term water quality variations is considered to be the central problem facing water quality administrators.

Managerial Applications of the Causal Analysis

When identification of the causal factors of estuarine water quality and the interactions between these factors has been accomplished, the managerial problems of controlling the factors may be considered. Many of the problems arise because there exists a multitude of alternative controls on the causal factors, each of which will accomplish a given improvement in quality. Those alternatives whose effects will be at least as good as desired are called "feasible alternatives. A manager is faced with three problems here:

1) What are the constraints which should determine what constitutes a "feasible" alternative?
2) Having specified the constraints, what alternatives are feasible?
3) Among the feasible alternatives, which one will result in the most efficient allocation of resources?

Although each of these will be discussed and exemplified later, some comments are in order here. The words "most efficient" in (3) do not necessarily mean "least expensive." If a strong acid solution were to enter the estuary, it would be more important to neutralize it quickly than cheaply. In this paper, the phrase "criterion of optimality" will be used to designate the measure of efficiency. In the same sense, an "optimal" feasible alternative is one which is at least as efficient as any other feasible alternative. It should be noted that in some situations, there may be several optimal alternatives; i. e. they are equally as good as one another and better than all the other feasible alternatives.

With this terminology, many of the problems can be defined abstractly as: What combinations of controls on the causal factors will have the desired effect on quality and be optimal? When a problem can be posed this way, it is likely that some of the techniques subsumed under "Operations Research" may be brought to bear on its analysis.

These mathematical and statistical techniques have, for the most part, been derived during the past ten to twenty-five years. Their application permits a more scientific approach to the making of decisions affecting situations over which some control can be exerted (i. e. "operational" problems.) First applied to military problems during World War II, Operations Research spread to industry during the fifties. Now all fields of engineering are confronted with highly complicated systems whose control is intended and, therefore, Operations Research is being applied throughout engineering. That estuarial systems are complex can be seen from the previous section of this paper. It would seem likely, therefore, that Operations Research techniques would be useful in problems of water quality control. This likelihood will be verified in this paper For an introduction to Operations Research literature and applications, see Churchman, et al in Selected References .

It is clear that undertaking a search for an optimal set of controls presumes the ability to measure which of any two feasible alternatives is more desireable (with respect to the criterion of optimality). If the criterion of optimality is net cost, for example, it is necessary to know the marginal change in net cost which would occur were any one of the controls varied (i.e. the marginal cost functions). As will be clear from an example letter, this data presently is lacking.

Strategy and Tactics

Many managerial problems of estuarial water quality control can be classified as either strategic or tactical in nature. The problem of strategy is the proper recognition of goals and the development of long-range programs. The tactical problems, on the other hand, arise from the inability to control long in advance and, therefore, to predict accurately, transient inputs to the estuary; these would include storm surges, increases in loads due to treatment plant breakdowns and oil spills.

The managerial problem that results from such inputs as these is tactical in nature: what is the optimal deployment of quality control resources that will countervail the effect of the undesireable input? It is logical to expect the criterion of optimality for a tactical policy to be consistent with the goals at which the long-range policies are aimed. Therefore, the strategic problems of estuarial water quality management will be considered before analyzing some tactical problems.

The specification of the criterion of optimality and the constraints on acceptable alternatives is equivalent to the selection of the goals at which the strategy of the quality control program will be directed. As explained in the Introduction, this choice of goals should be a response to the needs not only of the estuarine environment but also to the social, economic, political and technological environments of the estuarial region. The nature of the latter environments has a high rate of evolution and this causes a shift, over time, of the goals which are chosen. If a strategy of quality control is to be satisfactory, therefore, the criteria of optimality and the constraints must be flexible enough to adapt to continually changing environments.

Adaptive Control Policies

Up to this point, the descriptions of managerial strategy problems have assumed that the future quality of an estuary could be completely controlled by its manager. Actually, although the manager may have implemented his previous decisions, the quality will be affected by the vagaries of population growth, the evolution of industrial processes, in and out-migration from the estuary's region, redistribution of population densities within the region and other occurrences whose future magnitudes cannot be foretold. At future times, a manager may again have to make a decision because of the unexpected occurrences and the evolutionary nature of long-range goals.

An example of such a sequence may be provided by the location of the water treatment plant for a growing city located along an estuary. As the city grows, there may be an infusion of industries and expansion of existing ones. Their activities may increase the industrial waste load to the estuary while the municipality's waste discharge may also increase because of the larger population being served. Even if these effluents are all downstream from the water plant, tidal mixing will decrease the water quality at the plant's intake. This will cause higher operating expenses and a less desireable product. Consequently, the plant management is faced with the alternatives of staying put, moving up the estuary or moving out of the estuary. The choice ought not to be made, however, only on the basis of the present environment. The future evolution of the industrial and municipal pattern should be considered. A place further up the estuary may suffice, for the present, but will another relocation problem arise in the foreseeable future? Matters are further complicated because whatever action is taken will not only be a reaction to the present environment but will also help determine the future environment. Relocation of the plant to a rural area (along the

estuary), for instance, may make a public supply available there for the first time. This may accelerate urbanization of the area and its attendant waste material thus causing the original problem to arise once again.

Abstractly, this dilemma may be viewed as a sequence of decisions, each one of which affects subsequent estuarine quality but does not completely determine it. Associated with each decision is its (immediately known) cost of implementation. Furthermore, as time passes, the manager may learn more about the mechanics of water quality evolution and, therefore, reduce his uncertainty of the long-range effects of his decisions. The process that embodies this compounding of uncertainty, learning, sequential decisions and cost is called an "adaptive control process." A special (and sometimes more easily analyzed) case of such a process arises when the uncertainty regarding a decision's consequences is not explicitly included in the problem's formulation (and, therefore, no learning process is involved); this is called a "dynamic programming" problem. For further reading, see Bellman under footnote 2

"Static" Analysis: An Investment Problem

An adaptive control process is dynamic in the sense that time plays an important role. The computational difficulties of solving dynamic formulations, however, are often formidable. Instead of these, "static" problems (where no cognizance is taken of the temporal interactions of successive decisions) may be posed. Less data and less computation may be required but they are paid for with less realism.

These relative advantages will be seen from a static abstraction of the following problem:[5]

At several places in the estuary it is desired to increase the

---

[5]Sobel, M. J., "On the Management of the Quality of Natural Water Systems," Contributed Paper M-1, Meeting of the Operations Research Society of America at Cleveland, May 28, 1963.

equilibrium quality (e.g. mean DO) by at least specified minimum amounts. For this purpose, funds will be invested in the improvement or creation of various facilities along the estuary. The manager wishes to know the minimum total investment that it is necessary to make and how this total should be allocated among the alternative sites and facilities.

This problem can be posed as a linear programming problem, a mathematical model, for whose solution there are well known computational algorithms.[6] The data that are necessary for its solution are of two kinds;

(a) At each potential site or existing facility, one must know the costs associated with the range of alternative actions that can be taken there. This relationship is the marginal cost function at that site or facility.

(b) It is necessary to be able to predict how an action taken at any site or facility will affect the qualities at each of the places (in the estuary) whose improvement is sought. This information is embodied in the transfer functions which were introduced earlier.

In a particular situation in which the mean DO is to be improved, for example, suppose a potential facility is a municipal plant with primary treatment. It would be necessary to know the costs of removing 10%, 20%, 30%, etc. of the B.O.D. which is presently in the effluent. Furthermore, the effects of these alternative improvements on the quality at various places must be foretold. These functions (the costs and the effects of alternative improvements) are needed for each facility from which

---

[6] Gass, S. I., Linear Programming, Methods and Applications, (1958) New York: McGraw-Hill.

it is possible to affect an improvement in the estuary.

How an estuary transforms its inputs into its state vector is inferred in equation (1). The details of this transformation are in equations (7), (8) and (9) from which it is clear that the state in a particular section is the sum of the appropriate transformations of inputs into all sections, i. e.

$$c_j(t) = \sum_i f_{o_i} A_{ij}(\omega)\cos[\omega t + \theta_{ij}(\omega)] . \tag{9}$$

In the following analysis, the periods of the forcing functions will be regarded as indefinitely large. This leads to treatment of $\{f_i(t)\}$ as a set of constants. This assumption is reasonable in the context of a manager's investment problem. He is planning the construction of facilities which will be in operation many years in the future. Furthermore, he is interested in improving the annual mean quality rather than fluctuations about that mean during the year (problems of transient control will be analyzed in a later section of this paper). Accordingly, as T (the period of a forcing function) gets large, $\omega = 2\pi/T$ gets small and, in the limit, goes to zero. It can be shown [p. 5 of Solodovnikov, Selected References] that the phase shifts $\{\theta_{ij}(\omega)\}$ are arctangent functions whose arguments are zero when $\omega = 0$. Therefore, as $\omega \to 0$,

$$A_{ij}(\omega)\cos[\omega t + \theta_{ij}(\omega)] \to A_{ij}(0). \tag{12}$$

Insertion of (12) in (9) gives

$$c_j = \sum_i f_i A_{ij}(0) \tag{13}$$

where, for clarity, f_i denotes f_{o_i} and c_j denotes the constant value of $c_j(t)$ which is the resultant state in section j caused by the constants $\{f_i\}$.

There are costs associated with all changes made in the real world. In particular, there are costs associated with any particular set of improvements to the estuary, f_i. It is assumed that the cost of $f_1, f_2, \ldots, f_n$ = cost of f_1 + cost of f_2 + ... + cost of f_n. Let $V_i(f_i)$ denote the cost of f_i; then the net cost of a set of improved inputs is $\sum_i V_i(m_i)$. The manager would like to minimize $\sum_i V_i(m_i)$ while he obtains the necessary improvements in the estuary's quality. Generally, not all of the n sections need improvement; say that in sections 1, 2, ..., M ($M \leq n$) it is necessary to bring about improvements in the amounts $c_1, c_2, \ldots, c_M$.

For example, imagine a five section estuary in two of whose sections it is necessary to raise the annual mean DO from five to eight mg./l. and seven to eight mg./l. respectively. Say that there are treatment plants in three of the five sections. At each of the three plants, it is possible to construct and operate various kinds of additional equipment to decrease the B.O.D. which is now in its effluent. The cost of removing the B.O.D., however, will increase as the B.O.D. which remains (after the supplementary treatment) is reduced. In the light of these costs, a manager would seek the combination of actions at the three plants which would, at minimum total cost, increase the DO to at least as high as 8 mg./l. in the two sections of interest.

In this example, M=2, and c_1 and c_2 would correspond to the improvements sought:

$$c_1 = 8 - 5 = 3 \text{ mg./l.}$$

$$c_2 = 8 - 7 = 1 \text{ mg./l.}$$

Here, there are only three sections from which improvements can be made; say that these are sections, two, three and four (the sections need not be numbered consecutively; here, for example, the order of the five sections may be five,

four, two, three, one). Therefore, $f_1 = f_5 = 0$. The problem, now, is to find f_2, f_3 and f_4 so that $V_2(f_2) + V_3(f_3) + V_4(f_4)$ is a minimum while both

$$A_{21}(0)f_2 + A_{31}(0)f_3 + A_{41}(0)f_4 \geq 3$$

$$A_{22}(0)f_2 + A_{32}(0)f_3 + A_{42}(0)f_4 \geq 1.$$

Furthermore, it is reasonable to restrict $f_2 \geq 0$, $f_3 \geq 0$ and $f_4 \geq 0$ because inexpensively attained improvement in sections one and two are not sought at the expense of worse quality in the other sections. In other words, improvements in input quality (not total quality) are traded off between sections until the minimum total cost is found which maintains $c_1 = 3$ and $c_2 = 1$.

The example above may be formulated more generally as a search for values of $f_1, f_2 \ldots, f_n$ which will

$$\text{minimize} \quad \sum_i V_i(f_i)$$

subject to

$$\begin{aligned} A_{11}(0)f_1 + A_{21}(0)f_2 + \ldots + A_{n1}(0)f_n &\geq c_1 \\ A_{12}(0)f_1 + A_{22}(0)f_2 + \ldots + A_{n2}(0)f_n &\geq c_2 \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad &\vdots \\ A_{1M}(0)f_1 + A_{2M}(0)f_2 + \ldots + A_{nM}(0)f_n &\geq c_M \end{aligned} \tag{14}$$

and $\quad f_1 \geq 0,\ f_2 \geq 0,\ \ldots\ldots,\ f_n \geq 0.$

It may be possible to solve such problems as the one above by visual inspection if the cost functions are linear, there are only three or four unknowns and only two or three inequalities. If, as is more likely for existing estuaries, there are more alternatives and constraints and the cost

functions are non-linear (approximable by piece-wise linear functions) then a systematic search must be conducted for an optimal solution. Fortunately, considerable attention has been paid by mathematicians to the development of efficient algorithms to solve this kind of problem (see footnote 6). Note that its solution requires knowledge of $\{V_i(m_i)\}$, the marginal cost functions of improving the inputs into the various sections. Only occasional analysis, however, has been directed at their specifications.[7]

Two initial drawbacks to this formulation are (1) that respectable computational efforts are necessary to solve linear programming programs and (2) the marginal costs have to be determined. Nevertheless, for most problems of interest, computers can do the computation and the data exist from which the marginal costs can be constructed. If the static formulation was to be expanded to a dynamic programming problem, however, much more information would be needed. It would be necessary, for example, to know how investments in water quality control will affect the demography and industrial development of the estuarial region and how these outcomes, in turn, will affect future water quality.

The static version of the investment problem could be solved repeatedly for several different sets of incremental improvements in quality (or else a parametric linear programming problem could be formulated: see chapter 8 in the reference of footnote 6). Then the changes in optimal total investment could be related to the resultant incremental increases in quality. These relationships would constitute the marginal cost functions of quality improvement at the various locations which are of interest. Such knowledge would be

---

[7] Logan, J. A., W. D. Hatfield, G. S. Russel and W. R. Lynn "An Analysis of the Economics of Wastewater Treatment," J. of W.P.C.F., vol. 34, No. 9, pp. 860-992.

valuable when the goals of a quality control problem are being chosen.[8]

Problems of Tactics

In water quality management, the tactical problems may be regarded as the difficulties which are associated with attempts to control transient fluctuations of quality. Any body of water can experience unhappy variations of severe magnitude but brief duration. These could be caused, for example, by treatment plant breakdowns. Some causes which are peculiar to an estuary are storm-caused salinity intrusion and oil spills from tankers. The latter is a significant threat to many estuaries because oil tankers from overseas usually terminate their trips and transfer their cargoes while in estuarial waters. Other transient problems can arise from discharges of highly acidic or alkaline wastes. The discussion that follows will use the salinity intrusion problem to illustrate the formulation of transient control problems. Nevertheless, the methodology can be applied to transient problems in the control of alkalinity, acidity, petroleum wastes, storm water overflows or other wastes which bypass treatment.

As an indication of the relative magnitude of some transient fluctuations, the winds and salinity time variations are shown in Figure 7 for a recent condition on the Delaware estuary. Although the storm had a brief duration, its effect on salinity was notable. Had a manager been able to control some of the fresh water flows entering the estuary from its tributaries, the intrusion might have been reduced. There would be certain costs, however, which would be associated with releases from impoundments on the tributaries. These would be analogous to "opportunity costs" in classical economics. By using an extra

---

[8] McKean, R., Efficiency in Government Through Systems Analysis (1958), New York: John Wiley & Sons, pp. 43-44

amount of water for salinity control purposes when it is unneeded now for navigation power, etc., a penalty may be suffered in the future. At some later time, that is, it would have been used to satisfy a need that will, instead, be left wanting. Consequently, an optimal solution is one that is at least as inexpensive as any other feasible solution and whose patterns of flow releases sufficiently reduce intrusion. The problem faced by the manager, therefore, may be posed as:

> What combination of patterns of releases at the impoundments restricts salinity intrusion to specified amounts at various places at the minimum cost of accomplishment?

This problem's mathematical formulation leads to a programming problem which makes use of the weighting functions previously introduced under Forecasting Water Quality. The transfer functions used in the investment problem's analysis are frequency functions. In that form they are inconvenient for use when programming flow releases over short time periods. The weighting functions, the analogous transformations of inputs to the state vector in the time domain, are handier for these purposes. The transfer functions, $\{\phi_{ij}\}$, are related to the weighting functions by

$$(15) \qquad k_{ij}(t) = \int_{-\infty}^{\infty} \phi_{ij} \exp(\omega t \sqrt{-1}) d\omega, \qquad t > 0$$

where $k_{ij}(t)$ denotes the weighting function which corresponds to ϕ_{ij}. In the same way that $c_j(t)$ is a superpositioning of inputs transformed by transfer functions, the output $c_j(t)$ depends linearly on the values of $f_i(\tau)$, for all $\tau \leq t$, with a weight $k_{ij}(t-\tau)$.

For this reason, $k_{ij}(t)$ is called a weighting function and

$$(16) \qquad c_j(t) = \sum_{i=0}^{n} \int_{0}^{t} f_i(\tau) k_{ij}(t-\tau) d\tau$$

which is equivalent to equations (8) and (9). Figure 8 exhibits a weighting function for the Delaware estuary whose shape is typical of $k_{ii}(t)$, i. e. the input and response are in the same segment.

For purposes of transient control, the interval of time which is of interest (i.e. the "planning time horizon") is in the order of several weeks. A control policy that specifies rates of releases each day or two days will be sufficiently precise. Therefore, instead of looking at $c_j(t)$ as t varies continuously from 0 to some end time T, a sequence of values will be used: $c_j(0)$, $c_j(\Delta)$, $c_j(2\Delta)$, ..., $c_j(T\Delta)$ where Δ is the unit interval (e.g. 2 days) between points in time so $T\Delta$ equals T, the point in time after which there is no interest. Now (17) may be approximated by

$$(17) \qquad c_j(t\Delta) = c_j(t) = \sum_{q=1}^{Q} \sum_{v=0}^{t} k_{qj}(t-v) f_q(v).$$

Note that instead of the index i ranging over n sections as in (17), the index q is used in (18). q ranges over the Q points (e. g. impoundments) at which flow control can be exercised. Similarly, $f_q(v)$ is a controlled variation from the normal out-flow regime at the q-th point of control at time v, $0 \leq v \leq t$, $t = 0, 1, \ldots, T$.

There may be several places in the estuary at which it is desired to briefly reduce the salinity which would otherwise exist for a short time. Let there be M such places; and at the j-th of these let $c_j(0)$, $c_j(1)$, ..., $c_j(T)$ be the sequence of minimum salinity reductions which must be attained during the time interval of interest. A complete policy during this interval, therefore, would specify $f_1(0)$, $f_1(1)$, $f_1(2)$, ..., $f_1(T)$, $f_2(0)$, $f_2(1)$, ..., $f_2(T)$ ...., $f_Q(0)$, ..., $f_Q(T)$ such that the necessary salinity reductions occur. Such a set would

tell a manager how to govern the rate of release at each of the Q places from which flow can be regulated, for each point in time (at intervals of Δ) during the interval of length T.

In accordance with the preceding description, the problem in abstract form is to find values for the Q(T+1) variables

$$\begin{matrix} f_1(0), f_1(1), \ldots, f_1(T) \\ f_2(0), f_2(1), \ldots, f_2(T) \\ \vdots \\ f_Q(0), f_Q(1), \ldots, f_Q(T) \end{matrix}$$

to minimize $$\sum_{q=1}^{Q} V_q\left[f_q(0), f_q(1), \ldots, f_q(T)\right]$$

subject to $$\sum_{q=1}^{Q} \sum_{v=0}^{t} k_{qj}(t-v) f_q(v) \geq c_j(t)$$

and $$f_q(v) \geq 0$$

for $$q = 1, \ldots, Q;\ j = 1, \ldots, M;\ \text{and } t = 0, \ldots, T,$$

where $V_q\left[f_q(0), f_q(1), \ldots, f_q(T)\right]$ denotes the cost of the altered input at the q-th point of control.

Fortunately, the constraints (the inequalities) are linear in the unknowns, $\{f_q(t)\}$, but the net cost functional (the "objective function"), $V_q\left[f_q(0), f_q(1), \ldots, f_q(T)\right]$ need not necessarily be of the form $\sum_{t=0}^{T} V_{qt}^{*}\left[f_q(t)\right]$. If it is separable in this sense, then the formulation is that of a linear programming problem (assuming $V_q^{*}(\cdot)$ to be piece-wise linear). The costs of controlling the transient behaviour of a system, however, frequently are affected by the pattern of $f_q(0), f_q(1), \ldots, f_q(T)$. For example, the cost may depend on changes in the level of control from one point in time to the next and, therefore, may be of the sort $\sum_{t=0}^{T} V_{qt}^{*}\left[f_q(t)\right] + \sum_{t=1}^{T} V_{qt}^{**}\left[f_q(t+1) - f_q(t)\right]$.

Cost functions such as these arise in industrial inventory and production control problems for which various solution techniques have been devised.[9]

The full and immediate application of this elegant yet simple formulation may be prevented by several obstacles. As in the investment problem, equation (17), there does not exist an extensive bibliography which presents the cost functions in the form in which they are needed. Nevertheless, both the necessary economic theory and engineering cost analyses, exist; it remains to combine them. Another consideration in the use of (19) is the computational effort necessary to solve it. (19) is a static formulation of a dynamic problem (i. e. time is explicitly involved; it is the argument of $f_q(\cdot)$, the unknowns). It is possible, therefore, that a dynamical formulation would require less computation. This alternative has not yet been investigated.

Concluding Remarks

The preceding examples have attempted to demonstrate the significant managerial advantages of a detailed understanding of the estuarine environment. In the light of this understanding, many important water quality control problems can be postulated. Their formulations then are amenable to some methods of Operations Research. These mathematical techniques of optimization can find combinations of technological alternatives which are optimal in the sense that they specify the allocation of resources which is most efficient, subject to whatever constraints arise from the estuary's socio-political environment.

It is clear that the linear systems analysis of estuarial water quality

---

[9]Arrow, K. J., S. Karlin and H. Scarf, Studies in the Mathematical Theory of Inventory and Production (1958), Stanford Univ. Press. pp. 70-85

can be applied more widely than was done in this paper. There is an immediate generalization of all of the contents to straight run streams, lakes and impoundments. Indeed, from the viewpoint of the computations necessary to forecast and efficiently control their qualities, these environments are less complicated than estuaries.

Similarly, the formal structures for the managerial problems (17) and (19) are more general than the two examples from which they were derived. The derivations used the estuary's dynamical characteristics; these were embodied in the transfer function which resulted from the linear systems analysis. It was shown that the two problems' formulations are the means to obtain optimal solutions to two important classes of water quality management problems.

Selected References

Brown, B. M., The Mathematical Theory of Linear Systems, (1961) New York: Wiley

Churchman, C. W., R. L. Ackoff and E. L. Arnoff, Introduction to Operations Research, (1957) New York: Wiley

Dorfman, R., P. A. Samuelson and R. M. Solow, Linear Programming and Economic Analyses, (1958) New York: McGraw-Hill

Lynn, W., J. Logan and A. Charnes, "Systems Analysis for Planning Wastewater Treatment Plants," J. of W.P.C.F., Vol. 34, No. 6, p. 565

Solodovnikov, V. V., Introduction to the Statistical Dynamics of Automatic Systems, Dover Pub., New York, N. Y., xx+307 pp., 1960

FIGURE 1

AVERAGE DAILY DISSOLVED OXYGEN (mg./l.)

FOR DELAWARE ESTUARY

AT PIER #11 NORTH, PHILADELPHIA, PENNSYLVANIA

DEC. 1960 – NOV. 1961

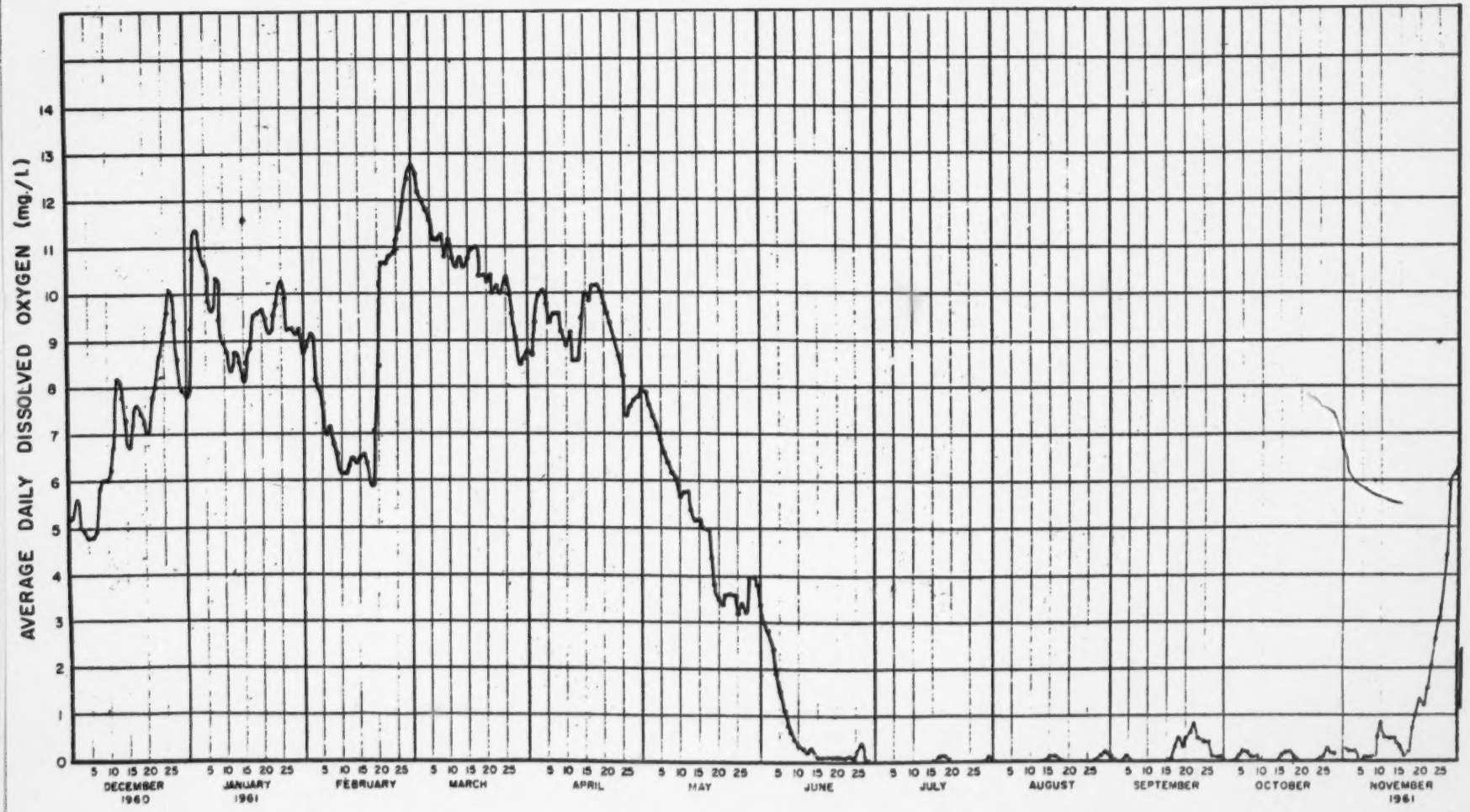


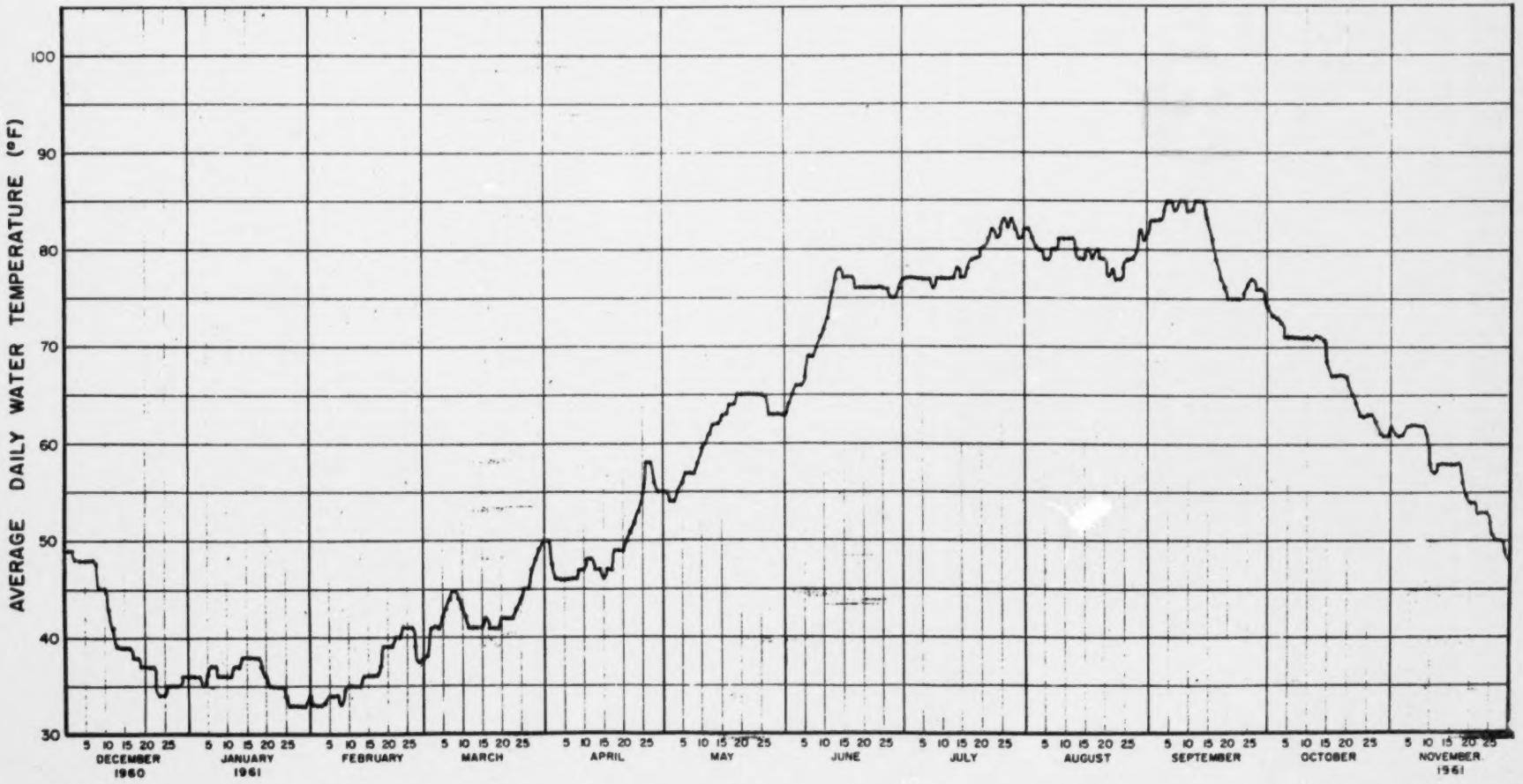
FIGURE 2

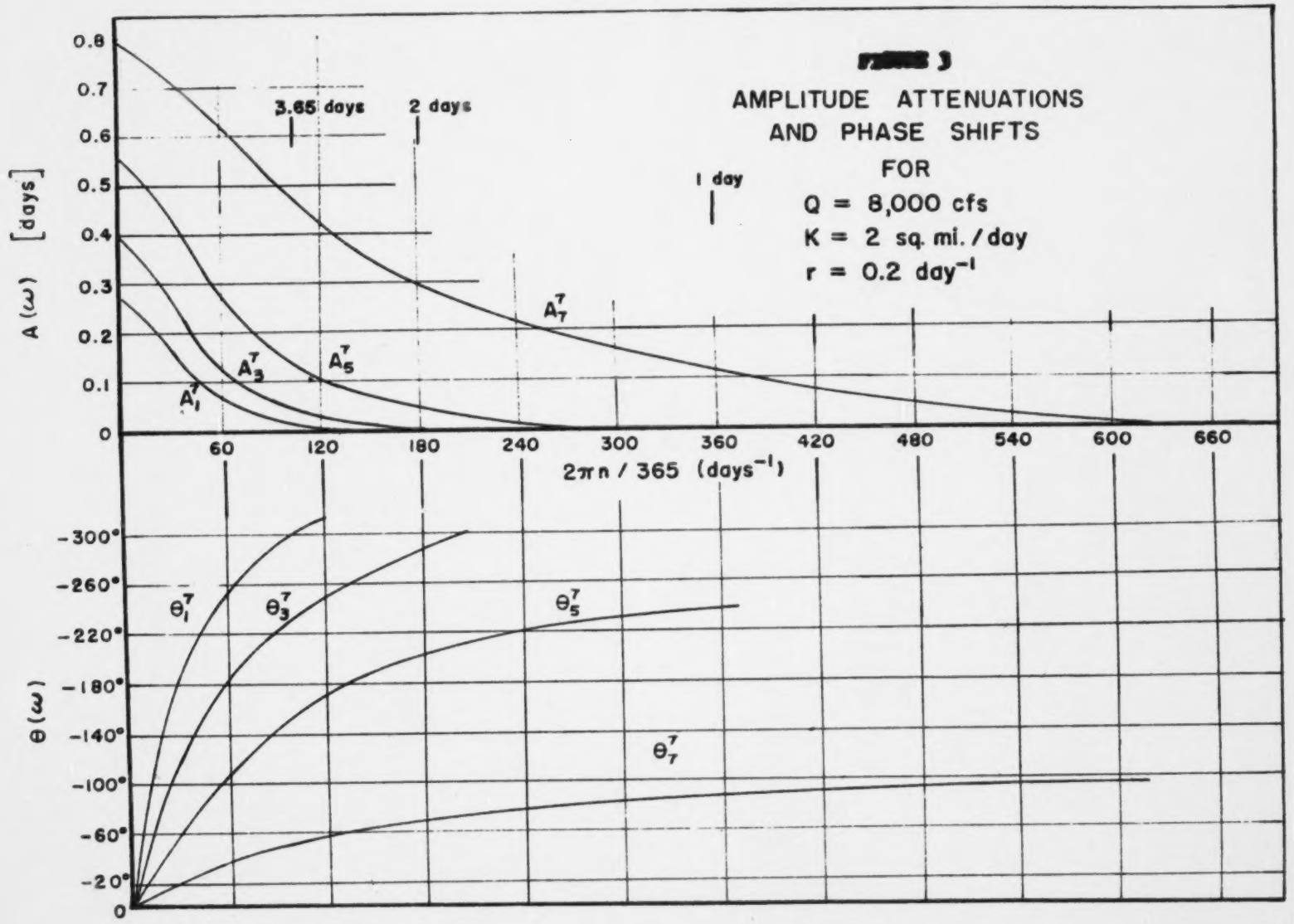
AVERAGE DAILY WATER TEMPERATURE (°F)

FOR DELAWARE ESTUARY

AT PIER # II NORTH, PHILADELPHIA, PENNSYLVANIA

DEC. 1960 – NOV. 1961




AMPLITUDE ATTENUATIONS
AND PHASE SHIFTS
FOR
Q = 8,000 cfs
K = 2 sq. mi. / day
r = 0.2 day⁻¹
1 day
3.65 days
2 days
A(ω) [days]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
A₁⁷
A₃⁷
A₅⁷
A₇⁷
60
120
180
240
300
360
420
480
540
600
660
2πn / 365 (days⁻¹)
θ(ω)
-300°
-260°
-220°
-180°
-140°
-100°
-60°
-20°
0
θ₁⁷
θ₃⁷
θ₅⁷
θ₇⁷

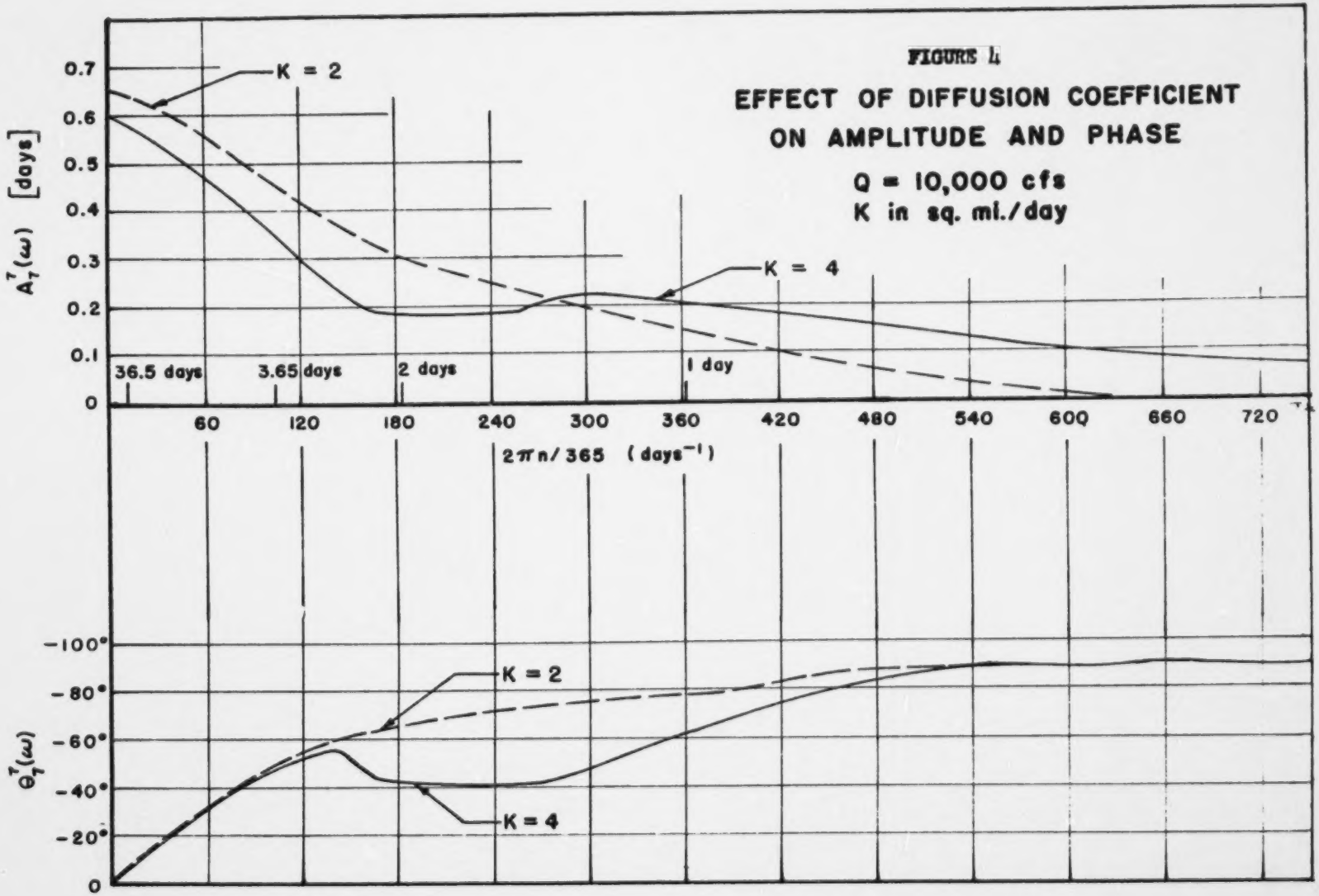


FIGURE 4
EFFECT OF DIFFUSION COEFFICIENT
ON AMPLITUDE AND PHASE
Q = 10,000 cfs
K in sq. mi./day
$A_T^Z(\omega)$ [days]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
K = 2
K = 4
36.5 days
3.65 days
2 days
1 day
60
120
180
240
300
360
420
480
540
600
660
720
$2\pi n/365$ (days^{-1})
$\theta_T^Z(\omega)$
-100°
-80°
-60°
-40°
-20°
0
K = 2
K = 4

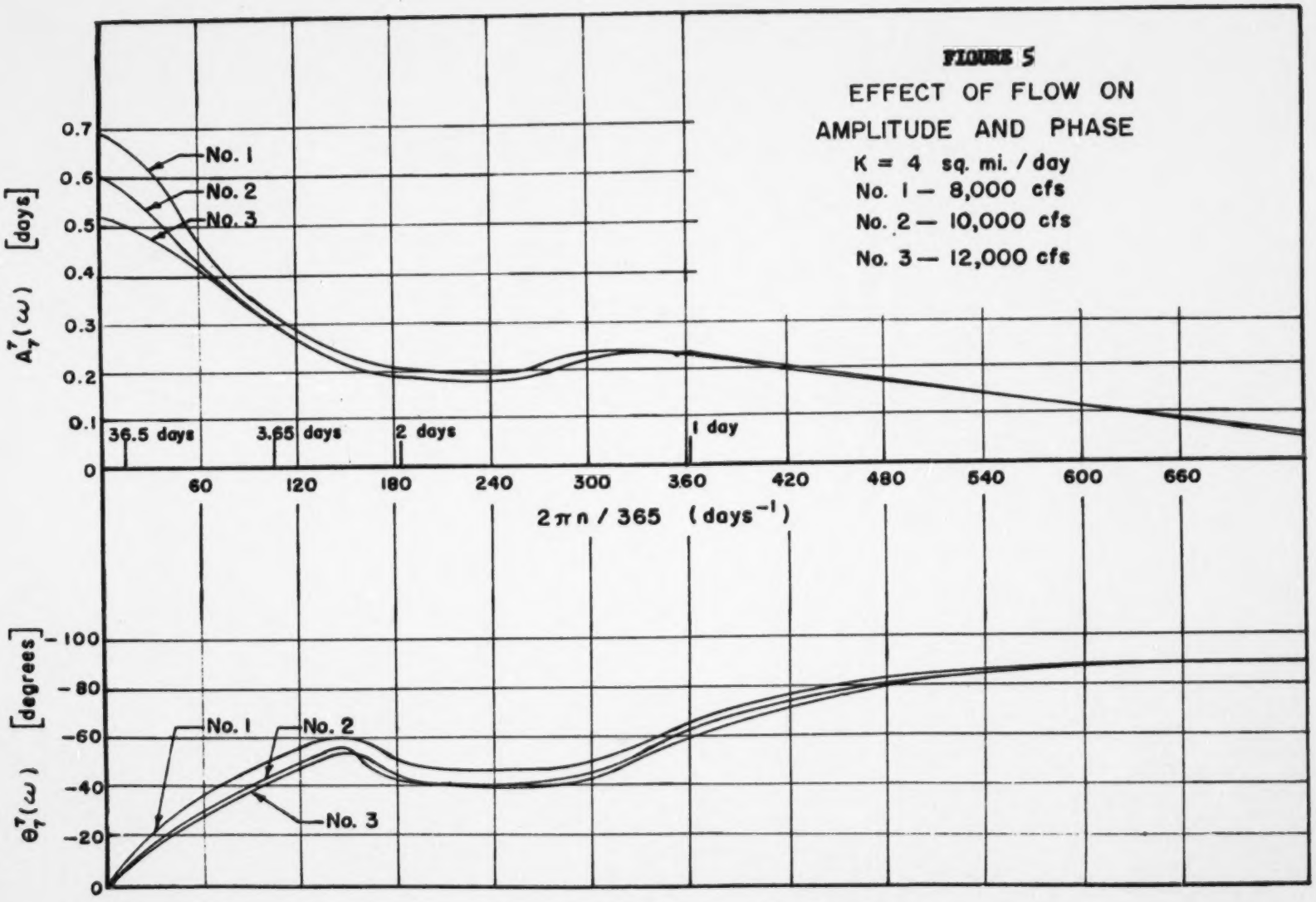


FIGURE 5
EFFECT OF FLOW ON
AMPLITUDE AND PHASE
K = 4 sq. mi. / day
No. 1 — 8,000 cfs
No. 2 — 10,000 cfs
No. 3 — 12,000 cfs
$A_7^T(\omega)$ [days]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
No. 1
No. 2
No. 3
36.5 days
3.65 days
2 days
1 day
60
120
180
240
300
360
420
480
540
600
660
$2\pi n / 365$ (days^{-1})
$\theta_7^T(\omega)$ [degrees]
-100
-80
-60
-40
-20
0
No. 1
No. 2
No. 3

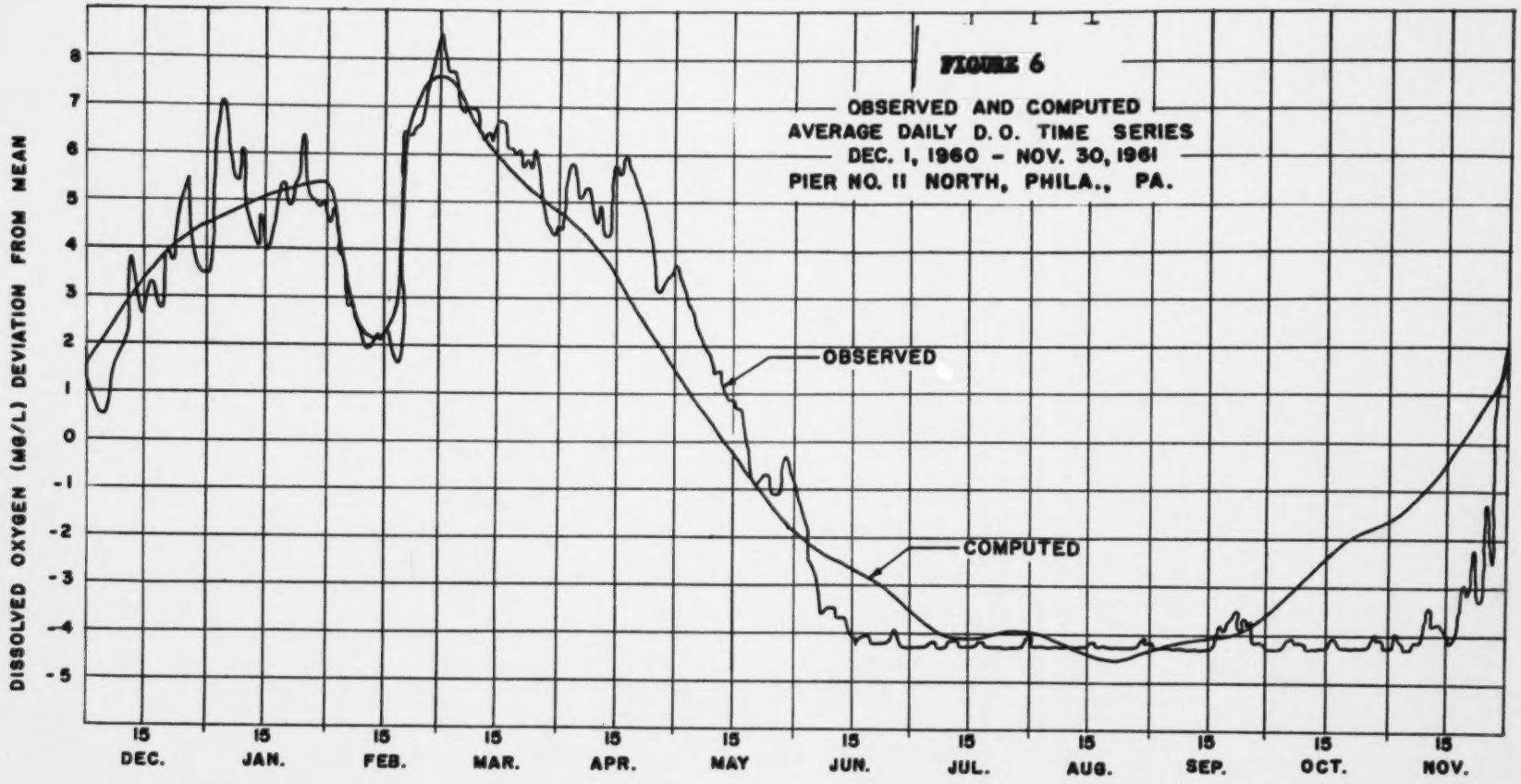


FIGURE 6
OBSERVED AND COMPUTED
AVERAGE DAILY D.O. TIME SERIES
DEC. 1, 1960 - NOV. 30, 1961
PIER NO. 11 NORTH, PHILA., PA.
OBSERVED
COMPUTED
DISSOLVED OXYGEN (MG/L) DEVIATION FROM MEAN
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
15 DEC.
15 JAN.
15 FEB.
15 MAR.
15 APR.
15 MAY
15 JUN.
15 JUL.
15 AUG.
15 SEP.
15 OCT.
15 NOV.

FIGURE 7

AVERAGE DAILY ULTIMATE BOD SINK (mg./l./day)

FROM NORTHEAST WORKS
PHILADELPHIA, PENNSYLVANIA
DEC. 1960 – NOV. 1961

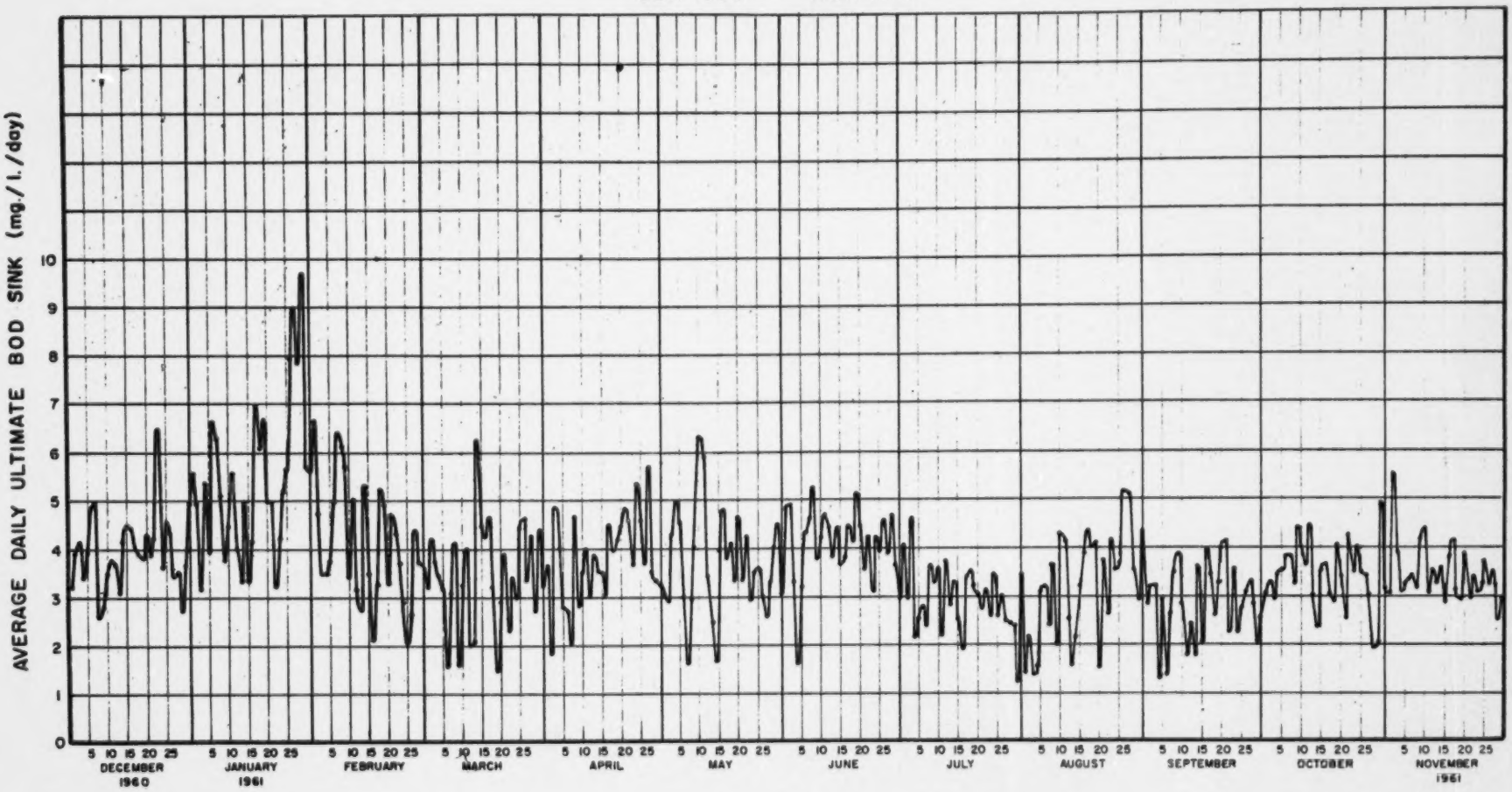
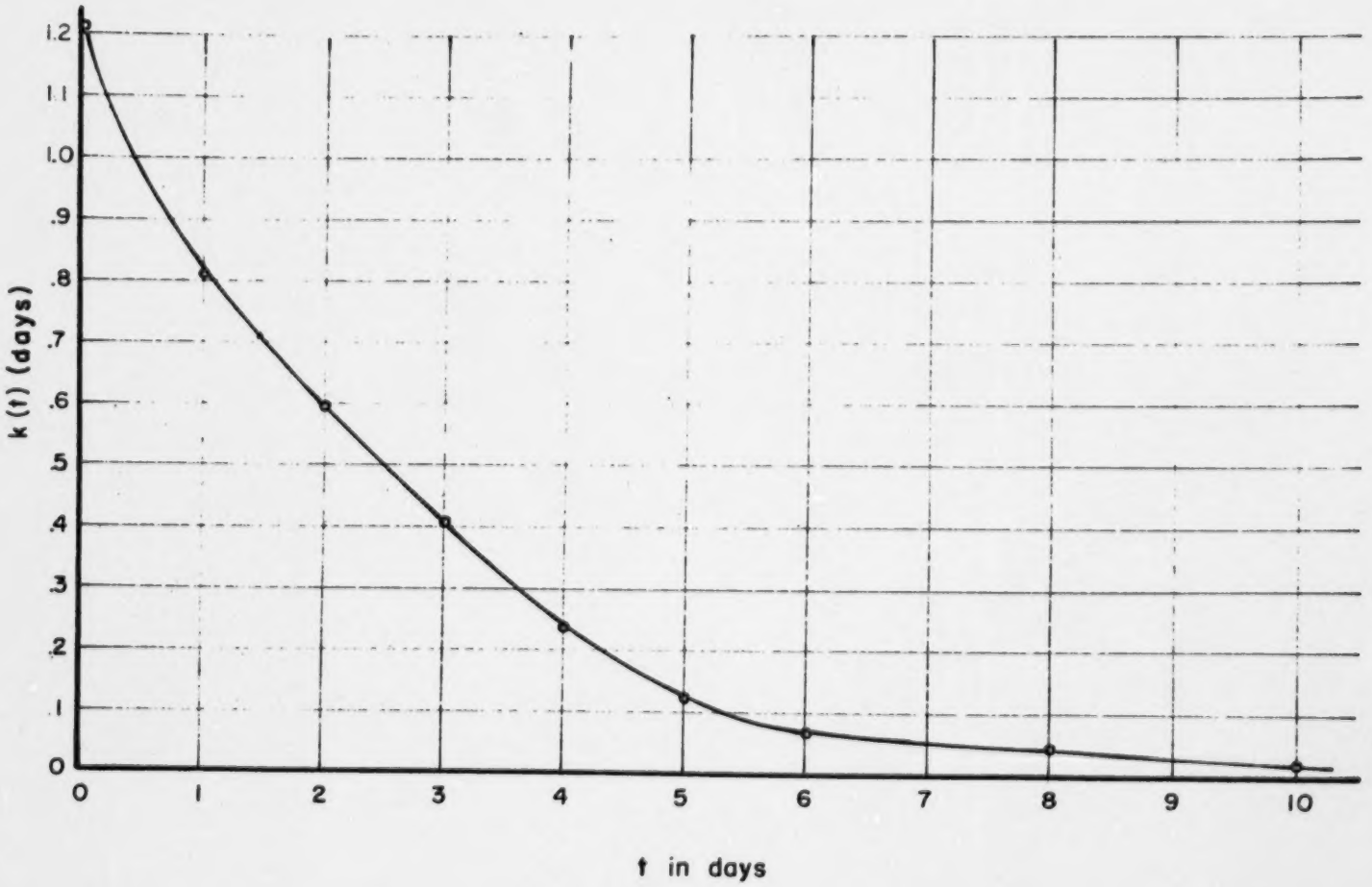
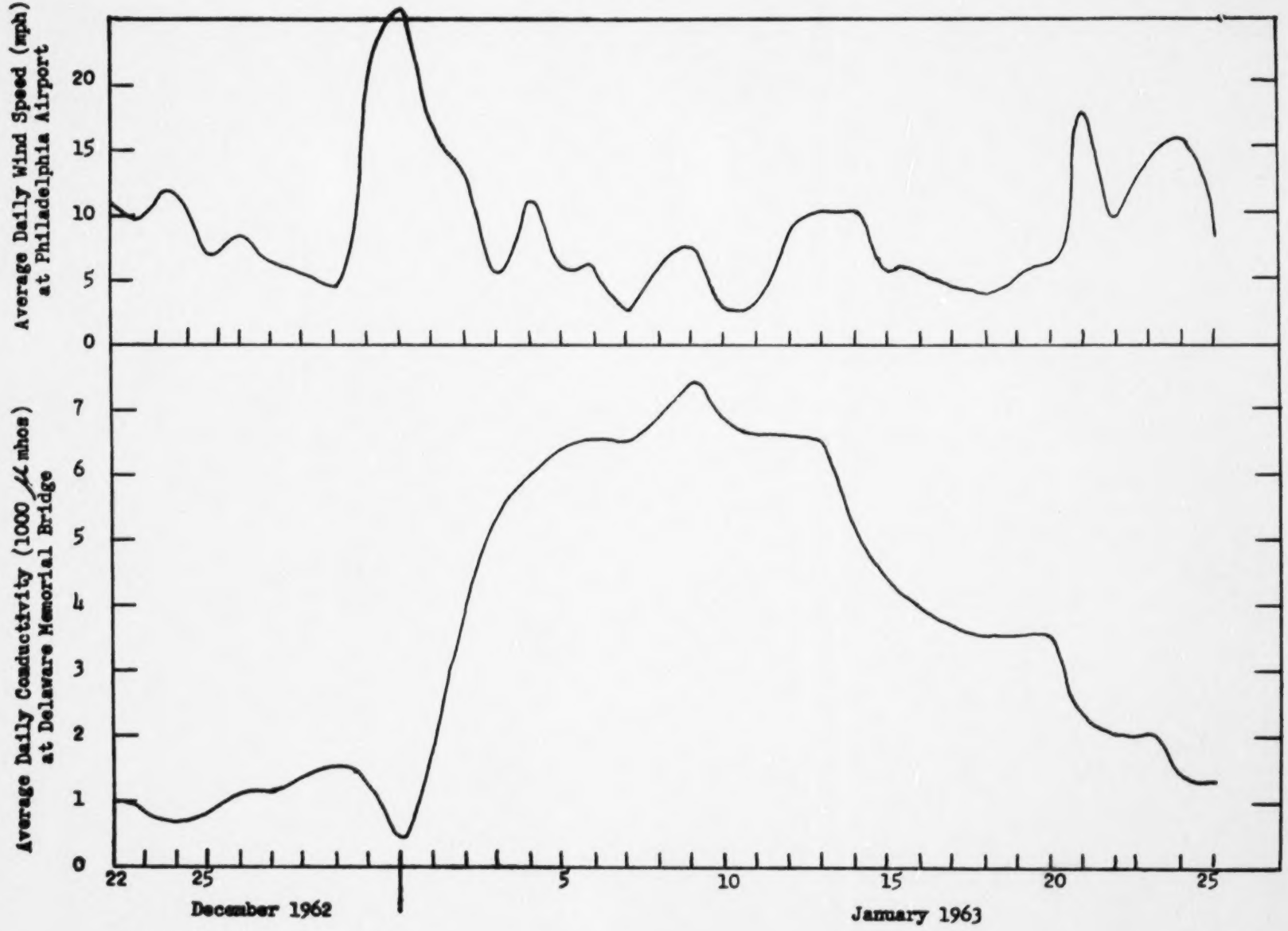


FIGURE 8

OVERALL IMPULSE RESPONSE FUNCTION k(t)



FI GURE 9



GPO 946-533